

Find the derivative.

$$\begin{aligned}
 1. \quad y &= \sin x & \frac{\cos x}{\sec^2 x} & 2. \quad y = e^x & e^x \\
 y &= \tan x & \frac{1}{\sqrt{1-x^2}} & y = 2^x & 2^x \cdot \ln 2 \\
 y &= \sin^{-1} x & & y = \cos x & -\sin x
 \end{aligned}$$
  

$$\begin{aligned}
 3. \quad y &= \ln x & \frac{1}{x} & 4. \quad y = \tan^{-1} x & \frac{1}{1+x^2} \\
 y &= x^3 - 8x^2 + 5x & 3x^2 - 16x + 5 & y = \cos^{-1} x & \frac{-1}{\sqrt{1-x^2}} \\
 y &= 3e^{4x} & 3e^{4x} \cdot 4 & f(x) = x^2 \sin(\pi x) & 2x \sin(\pi x) + x^2 \pi \cos(\pi x)
 \end{aligned}$$

## Opener

Let  $f$  and  $g$  be functions that are differentiable everywhere. If  $g$  is the inverse function of  $f$  and if  $g(-2) = 5$  and  $f'(5) = -\frac{1}{2}$ , then  $g'(-2) =$

(-2, 5)

- (A) 2      (B)  $\frac{1}{2}$       (C)  $\frac{1}{5}$       (D)  $-\frac{1}{5}$       (E) -2

$$\begin{aligned}
 g &(-2, 5) & -2 \\
 f &(5, -2) & -\frac{1}{2}
 \end{aligned}$$

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If  $y = \frac{\ln x}{x}$ , then  $\frac{dy}{dx} =$

(A)  $\frac{1}{x}$       (B)  $\frac{1}{x^2}$       (C)  $\frac{\ln x - 1}{x^2}$       (D)  $\frac{1 - \ln x}{x^2}$       (E)  $\frac{1 + \ln x}{x^2}$

$$\frac{f' \cdot g - f \cdot g'}{g^2}$$

Simplify the expression using properties of exponents.

$$\begin{aligned}
 1) \quad \ln(e^{\tan x}) & \tan x \\
 2) \quad \log_2(8^{x-5}) & = x \quad 2^x = 2^{3(x-5)} \\
 & 2^x = 8^{x-5} \quad x = 3x - 15 \\
 & \cdot 2^{x-15} \\
 3) \quad 3 \ln x - \ln 3x + \ln(12x^2) & x = 7.5 \\
 & \ln\left(\frac{x^3}{3x} \cdot 12x^2\right) = \ln(4x^4) \\
 4) \quad \ln(x^2 - 4) - \ln(x+2) & \ln\left(\frac{x^2 - 4}{x+2}\right) = \ln(x-2)
 \end{aligned}$$

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## 3-7 Implicit Differentiation

### Learning Objectives:

I can calculate the derivatives of implicitly defined function.

I can calculate the second and higher order derivatives of implicitly defined functions.

I can write the tangent and normal lines to implicitly defined functions.

**Implicit differentiation** is used whenever you need to find a rate of change (derivative) and the relation cannot be solved for  $y$  like with the equation:

$$x^3 y^2 - \cos y \cdot \ln x + e^x \sec^{-1} y = \sqrt{y^5 x^3}$$

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Ex1. Find the derivative of

$$\begin{aligned}x^2 + 3x + 2y + y^2 &= 0 \\2x + 3 + 2 \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} &= 0 \\2x + 3 + 2 \cdot y' + 2y \cdot y' &= 0 \\2y' + 2y \cdot y' &= -2x - 3 \\y'(2+2y) &= -2x - 3 \\y' &= \frac{-2x-3}{2+2y}\end{aligned}$$

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Ex2. Find the derivative of

$$\begin{aligned}\frac{(x-3)^2}{25} + \frac{(y+1)^2}{9} &= 1 \\\frac{1}{25}(x-3)^2 + \frac{1}{9}(y+1)^2 &= 1 \\\frac{1}{25} \cdot 2(x-3) \cdot 1 + \frac{1}{9} \cdot 2(y+1) \cdot 1y' &= 0 \\\frac{2}{25}(x-3) + \frac{2}{9}(y+1) \cdot y' &= 0 \\\frac{\frac{2}{9}(y+1) \cdot y'}{\frac{2}{9}(y+1)} &= \frac{-\frac{2}{25}(x-3)}{\frac{2}{9}(y+1)}\end{aligned}$$

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Ex3. Write the equation of the tangent line to the curve at the point (1,2)

$$\begin{aligned}x^2 + 3xy + y^2 &= 11 & f = 3x & g = y \\2x + 3xy' + 3y + 2y \cdot y' &= 0 & f' = 3 & g' = 1 \cdot y' \\2x + 3y' + 6 + 4y' &= 0 & (y-2) &= -\frac{8}{7}(x-1) \\8 + 7y' &= 0 & 7y' &= -8 \\y' &= -\frac{8}{7}\end{aligned}$$

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Ex4. Write the equation of the normal line to the curve at the point (1,2)  
(same curve that was in Ex3)

$$\begin{aligned}x^2 + 3xy + y^2 &= 11 \\2x + 3xy' + 3y + 2y \cdot y' &= 0 \\y - 2 &= \frac{7}{8}(x-1)\end{aligned}$$

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Ex5. Find the second derivative of

$$\begin{aligned}x^2 + 3x + 2y + y^2 &= 0 \\y' &= \frac{-2x-3}{2+2y} \\\frac{d^2y}{dx^2} = y'' &= \frac{-2(2+2y) - (-2x-3)2 \cdot y'}{(2+2y)^2} \\&= \frac{-2(2+2y) - (-2x-3) \cdot 2 \left(\frac{-2x-3}{2+2y}\right)}{(2+2y)^2}\end{aligned}$$

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### Homework

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